Midterm Exam

Statistical Physics

Tuesday December 13, 2016
09:00-11:00

Read these instructions carefully before making the exam!

- **Write your name and student number on *every* sheet.**
- ***Make sure to write readable for other people than yourself. Points will NOT be given for answers in illegible writing.***
- ***Language*; your answers have to be in English.**
- **Use a *separate* sheet for each problem.**
- **Use of a (graphing) calculator is allowed.**
- **This exam consists of 3 problems.**
- **The weight of the problems is Problem 1 (P1=27 pts); Problem 2 (P2=27 pts); Problem 3 (P3=36 pts). Weights of the various subproblems are indicated at the beginning of each problem.**
- **The grade of the midterm exam is calculated as (P1+P2+P3+10)/10.**
- **For all problems you have to write down your arguments and the intermediate steps in your calculation, *else the answer will be considered as incomplete and points will be deducted.***

PROBLEM 1 *Score: a+b+c+d+e =5+5+5+5+7=27*

The equation of state of one mole of an ideal gas is given by $PV = RT$ and its energy is given by $E = \frac{3}{2}RT$.

One mole of a perfect gas initially at temperature T_0 expands *reversibly* from volume V_0 to $2V_0$. Calculate the following quantities and express your answers in R and T_0.

a) The work W_1 done ***by*** the gas when the expansion takes places at constant temperature.
b) The heat Q_1 absorbed ***by*** the gas when the expansion takes places at constant temperature.
c) The work W_2 done ***by*** the gas when the expansion takes places at constant pressure.
d) The heat Q_2 absorbed ***by*** the gas when the expansion takes places at constant pressure.

The molar heat capacities at constant volume C_V and at constant pressure C_P are for a perfect gas related by $C_p = C_V + R$.

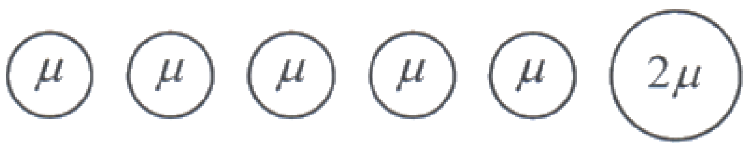
e) Prove this relation.

PROBLEM 2 *Score: a+b+c+d=6+7+7+7=27*

One of the fundaments of equilibrium statistical physics is the postulate of equal *a priority* probabilities.

a) Explain this postulate.

An isolated system consists of a fixed row of 6 spins, 5 spins have a magnetic moment μ and one of the spins has a magnetic moment 2μ (see Figure). Each spin can occupy two states: 'up' and 'down' relative to an external magnetic field. For the first five spins the energy levels are $-\varepsilon$ for an 'up' state and ε for a 'down' state. The sixth spin has energy levels -2ε for an 'up' state and 2ε for a 'down' state.



It is given that the total energy of the isolated system is -3ε.

b) Calculate the entropy of the system.
c) Calculate the probability that the sixth spin is in the 'up' state
d) Calculate the mean number of spins in the 'up' state.

PROBLEM 3 *Score: a+b+c+d =9+9+9+9=36*

According to quantum theory, the energy levels of a harmonic oscillator are given by $\varepsilon_n = \hbar\omega\left(n + \frac{1}{2}\right)$; $n = 0,1,2\cdots$, with ω the angular frequency of the oscillator. Assume that such an oscillator is in equilibrium with a heat bath at temperature T.

a) Show that the partition function of this oscillator is given by,

$$Z_1 = \frac{1}{2\sinh y} \quad \text{with } y = \frac{\hbar\omega}{2kT}$$

b) Prove that the mean energy $\bar{\varepsilon}$ of the oscillator is given by:

$$\bar{\varepsilon} = \frac{1}{2}\hbar\omega\frac{\cosh y}{\sinh y}$$

Now consider a 2-dimensional solid that consists of N atoms. Each atom can oscillate in two independent directions and all these oscillations can be considered harmonic and have the same angular frequency ω (thus, Einstein's theory can be applied!).

c) Show that the heat capacity C_V of this 2-dimensional solid can be written as:

$$C_V = 2Nk\left(\frac{y}{\sinh y}\right)^2$$

d) Calculate C_V in the low ($T \to 0$) and high ($T \to \infty$) temperature limit. Discuss the low temperature limit in relation to the third law of thermodynamics and discuss the high temperature limit in relation to the law of Dulong and Petit for 3D solids.

Hyperbolic functions:	
$\cosh x = \frac{e^x + e^{-x}}{2}$	$\frac{\partial}{\partial x}\cosh x = \sinh x$
$\sinh x = \frac{e^x - e^{-x}}{2}$	$\frac{\partial}{\partial x}\sinh x = \cosh x$
$\cosh^2 x - \sinh^2 x = 1$	